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Mathematics education for students with Autism Spectrum Disorder: Where are we now?*

Abstract. Individuals diagnosed with Autism Spectrum Disorder (ASD), the fastest growing disability group, exhibit varying degrees of intellectual ability. Students with ASD are increasingly held accountable to academic standards comparable to their peers. Applied Behavior Analysis (ABA) is widely considered best practice for supporting these students. Twenty-six single-case design (SCD) mathematics classroom teaching interventions, conducted with students diagnosed with ASD, were systematically located and reviewed in detail. Most interventions were conducted in special education mathematics classrooms involving low ability students. Interventions typically targeted simple mathematics skills, and a paucity of research addressing more complex mathematical skills was noted. Elsewhere in the literature, teachers who have students with ASD in their classrooms reported having received no autism training, and described subsequent stress and potential to burnout as a result. A need for future research with high ability mathematics students is observed, and the relevance of a values paradigm approach is proposed.

1. Introduction

The field of autism attracts researchers from many different scientific and scholarly fields – including mathematics education – utilising various research paradigms to investigate the topic. Autism research has been approached with numerous aims and methods ranging from identifying possible causes, to promoting optimal outcomes for individuals with a diagnosis. A recent study conducted by Iuculano et al. (2014) used cognitive assessments and functional magnetic resonance imaging (fMRI) brain scans to investigate the mathematical ability in children aged 7–12 years. Single-digit addition problem solving was examined. The researchers reported the use of sophisticated decomposition strategies, and that better numerical

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problem solving ability was demonstrated by the children with High Functioning (HF) Autism Spectrum Disorder (ASD) when they were compared to matched control group students (Iuculano et al., 2014).

Individuals with ASD exhibit varying degrees of intellectual ability and delay in social communication. ASD is currently estimated to occur in one in every 68 births (1.47%) (CDC, 2014). Global epidemiological studies have suggested that ASD prevalence is not influenced by race or immigrant status (Fombonne, 2005). However, there is no standardised methodology to conduct epidemiological surveys, and this variability is reflected in the literature (Fombonne et al., 2016). Kim, et al. (2011), for example, found that as many as one in every 38 children (2.64%) in South Korea may be diagnosed with ASD.

To date, no biological marker exists for ASD. Diagnosis is made on the basis of clinical observations, described most recently in the DSM-5 (2013). Current diagnosis of ASD is defined in two behavioral domains both presenting in early childhood: persistent impairment in reciprocal social communication and interaction; and restricted, repetitive patterns of behavior. Sub categories including Asperger's syndrome (AS) and Pervasive Developmental Disorder – Not Otherwise Specified (PDD-NOS) that existed in earlier DSM versions, and were considered to describe higher functioning individuals, have been dropped from the DSM-5.

For many, interest persists in distinguishing high and low functioning students. Criteria for the classification of students diagnosed with ASD as either High- or Low-Functioning have been described in the literature (Carr et al., 2014; Carr, 2016). Author adopted traditional cut-off points of the Childhood Autism Rating Scale (CARS) and Wechsler Intelligence Scale for Children (Dickerson et al., 2011). A classification of High Functioning Autism (HFA) required an IQ score of 80 or higher, the use of functional language, a CARS score less than 30, or an original researcher report of high functioning ability. A classification of Low Functioning Autism (LFA) required an IQ score of less than 80, restricted communication/language or life skills, a CARS score equal to or greater than 30, or an original researcher report of low functioning ability.

The United States Department of Education (USDOE) recently reported that students with ASD represent the fastest growing disability group in schools, accounting for 7.6% of students receiving special education services under the Individuals with Disabilities Education Act (IDEA) (USDOE, 2014). Today in Australia, the Disability Discrimination Act (DDA) stipulates that individuals with a disability have equal rights with the rest of the community regarding education and training (Disability Standards for Education, 2005). Similarly, legislation such as IDEA in the USA stipulates that individuals with disabilities have the right to attend their local public school and to receive government funded support as necessary.

At the same time, individuals with ASD are increasingly held accountable to academic standards comparable to their peers (Schaefer-Whitby, 2009). In the U.S. the Common Core State Standards in mathematics (CCSS-M) (National Govenors Association Center for Best Practices & Council of Chief State School

Officers, 2010) are designed to improve postsecondary outcomes for individuals with disabilities and as such unprecedented increases in mathematics performance is expected for all students, including those with a disability.

Varying reports on the mathematical abilities of students with ASD are found in the literature. Some researchers have described mathematical talent in individuals with autism (McMullen, 2000; Ward, Alar, 2000). Conversely, based upon low numerical operations subtest scores on the Wechsler Individual Achievement Test (WIAT), Griswold et al., (2002) reported that students with ASD had mathematical deficits. Chiang and Lin (2007) conducted a review of the literature for students with Asperger's syndrome (AS) or HF autism and reported that the majority of students in their sample demonstrated average mathematical ability when compared to the normed population. Chiang and Lin also reported that some individuals with AS/HF autism have mathematical giftedness.

2. Identifying Evidence Based Best Practice for Educating Students with ASD

Many treatment approaches exist for supporting students with ASD, however the standard of scientific proof that has been demonstrated by behavioral interventions using Applied Behavior Analysis (ABA) is generally agreed upon as constituting best practice for this population (Foxx, 2008). Single-case design (SCD) research has been adopted by the field to account for heterogeneity present amongst individuals with ASD, and has added in a cumulative manner to the identification of best practice for children with autism presented in the National Standards Report prepared by the National Autism Center (NSR, 2009).

Sigafoos and Schlosser (2008) reported that Applied Behavior Analysis (ABA) based treatments are considered to be the most consistently effective approach for educating children diagnosed with ASD. Over 1,000 peer-reviewed articles, based on principles of ABA, describe scientifically robust success with individuals of all ages (Foxx, 2008). Foxx reported that no other education treatment approach to autism meets the standards of scientific proof that are met by ABA, nor are there any other scientifically valid treatments that produce similar treatment, educational or outcome results.

ABA is widely used as a behavior modification approach to teach desirable behaviors and/or to extinguish problematic behaviors. The field of ABA that evolved from the early work of B. F. Skinner, has not been without controversy. In response to punitive misuse of practices ranging from shock treatment to solitary confinement, Skinner explained his original intention of the term behavior modification was "the management of human behavior through contingencies of positive reinforcement specifically designed to replace the punitive techniques that are commonly observed in prisons and used by parents, teachers, employers and others" (Skinner, 1974, p. 813).

In his paper addressing issues related to the legal regulation of ABA, Friedman (1975) investigated several contributing factors associated with public confusion and concern with ABA. Using the DSM language of that era, individuals with ASD were classified as mentally retarded and institutionalisation was common practice. A patients' right to treatment and their right to refuse treatment were acknowledged, with Friedman noting that beyond an institutional setting ABA involves a negotiated contract between the therapist and patient specifying mutually agreed upon goals and procedures. Friedman explained that many practices from a variety of fields that resulted in behavior modification, including abuses by psychosurgeons or psychopharmacologists, were erroneously viewed by the public as ABA. Additionally, it was reported that many untrained practitioners claimed to be working within the field of ABA, and had been conducting programs that were highly abusive and violative of the dignity and rights of institutionalised patients.

While the use of ABA to develop basic life skills was not contentious, Friedman highlighted that modification of complex behavioral repertoires may affect changes in attitudes or personality. Friedman noted that the latter may be criticised for curtailing freedom of choice and manipulation of human character. While ABA procedures had demonstrated effectiveness in controlled settings, researchers of the 1970's had not demonstrated that behavior change could generalise to real world settings. However, since the 1980's ABA has been applied in naturalistic settings beyond the traditional clinics. In the late 1980's Koegel and colleagues drew upon motivational literature to develop their Pivotal Response Training (PRT) techniques, and reported that students were able to generalise skills beyond the training setting as well as to master proficiency independently with untrained behaviors (Koegel, Koegel, Camarata, 2010).

Using the widely accepted and most current definition, Cooper, Heron and Heward (2007) have defined Applied Behavior Analysis (ABA) as the science in which tactics derived from the principles of behavior are applied systematically to improve socially significant behavior and experimentation is used to identify the variables responsible for behavior change (p. 20).

While intervention packages based upon ABA vary according the particular needs of an individual, they share similar features. Interventions aim to increase or decrease a target behavior. Data is collected using either frequency or interval counts of target behavior across all stages of intervention. A teaching technique is included eg. discrete trial teaching, incidental teaching, video (self) modelling, iPad instruction. Interventions often include prompting – most-to-least if teaching a new skill or least-to-most if skill exists, and may include (self) recording (self) recording of behavior. Ideally, the participant will receive appropriate positive reinforcement when achieving the desired behavior, and finally the intervention supports will be faded as appropriate behavior is mastered.

With increasing numbers of children with ASD receiving ABA therapy during their early childhood years, and an emphasis on least restrictive classroom placement, increased numbers of students with ASD are anticipated in mainstream classrooms in many developed countries. However, how prepared and ready are teachers and peers of students with ASD, both psychologically and emotionally? How do we ensure that the provision of access to mainstream school education for students with ASD is complemented with supportive dispositions, knowledge and skills informed by evidence-based best practice for the different people they will interact with on a nearly day-to-day basis?

In many locations around the world ABA based teaching is not always an option for students with ASD. An absence of service providers, geographic isolation, barriers to funding, and lengthy waiting lists may contribute to this scenario. Furthermore, while ABA interventions are guided by scientific principles, it has been acknowledged that this alone may not always be sufficient to guide decision making (Ruiz, Roche, 2007). Ruiz and Roche suggested that on occasion, values may function as a guide to action, and play and important role when ethical quandaries emerge. In the context of mathematics teaching and learning, such conflicts may arise as a result of limited resources, student cognitive ability, or the subsequent student and/or teacher frustration.

3. Purpose of This Review

Growing concern over the state of mathematics education for individuals with ASD is evident in the broader educational psychology literature. Single-case research designs (SCD) are widely used in special education research, as these designs are better suited to examine the effects of individualised interventions (Horner et al., 2005). Accordingly, to address this concern the purpose of this current literature review is to identify research findings from empirical mathematics studies that present best-evidence practice for individuals on the autism spectrum. A values paradigm has also been explored as a potential method to advance the understanding of the application of best practice for students with ASD in mathematics classrooms. Subsequently a research agenda will be proposed to inform future research and data collection that will give students on the autism spectrum a voice, as an important step towards improving mathematics pedagogy.

4. Method

A systematic literature search was conducted in the psycINFO data base. The search terms "autism*", "Asperger*" and "PDD-NOS" were each combined with the search term "math*". The search was restricted to peer reviewed, English language publications prior to December, 2016. No age limitations, or classroom setting restrictions were imposed.

The search identified two systematic literature reviews of mathematics interventions for students with a formal diagnosis of ASD, both recently published (King, Lemons, Davidson, 2016; Gevarter et al., 2016). The data sets of both reviews were based solely upon SCD intervention research. Accordingly, both reviews were retained for further analysis.

5. Findings

Studies included in the published reviews of King et al. (2016), and Gevetar et al. (2016) were compared. Table 1 summarises the original author publications included in each of the 2016 reviews.

Table 1

Empirical Intervention Research on Mathematics and Students with Autism Spectrum Disorder

Author	Gevarter et al. (2016)	King et al. (2016)	High/Low Functioning	Classroom Setting	Age (Years)	Grade	Teacher	Targeted Skills
Adcock & Cuvo (2009)		X	Dylan: High Liam: High Eli: Low	General Education; Intervention conducted in school therapy room	7 8 10	1st 2nd 3rd	All students had individual aide	Subtraction of 1 digit from 2 digits; Multiplication of 1 digit by 4 or 5; Telling time to 45 mins pas hour, addition of 2 digits to 1 digit
Akmanoglu & Batu (2004)	X		Omer: Low Baris: Low Serap: Low	Special education; Special education; Special education; All interventions in segregated setting	17 12 6	n/a n/a n/a	3 years experience teaching students with autism; masters in special education	Naming or successfully pointing to numeral on card
Banda & Kubina (2010)	X		Brad: High	General education; Intervention in resource room	13	n/a	General education teacher requested specialist support from university	Addition of 3 digit and 3 digit; missing addend problems
Bouck, Satsangi, Doughty & Courtney (2014)	X		Details not provided	Private non-profit clinic; ABA based therapy	6 7 10	1st Kinder garten 1st	ABA therapist at autism clinic	Subtraction problems using virtual and concrete manipulatives

Burton, Anderson, Prater & Dyches (2013)	X		Joey: High Will: Low Ryan: Low	Self-contained special education classrooms	13 14 15	4th 3rd 3rd	Classroom contained 1 licensed special educator, 2 paraeducators, 1 paraeducator for specific student	Read story problem, identify cost of item, estimate smallest number of bills required, estimate and calculate exact change, use cash register to tender exact change using fewest amount of bills and coins
Cihak & Frost (2008)	X	X	Travis: Low Ivy: Low Gina: Low	Special education resource classroom	7 7 8	Elementary grade n/a	2 special education teachers, both had at least 3 years experience teaching students with multiple and severe disabilities	Single digit addition problems, taught using both a number line and touch points
Cihak & Grim (2008)	X	X	John: Low Mary: Low Tom: Low Diane: Low	High school resource classroom; Department store community setting	16 16 17 15	High school students, grade n/a	Special education teachers, background n/a	Purchasing skills using counting on strategy (eg if price was \$17.21 student expected to tender \$18.00 using 1 \$10, 1 \$5, 3 \$1; tendering \$20 considered incorrect
Cihak, Wright & Ayers (2010)	X		Adam: High Jordan: High Richard: High	General education classroom	11 11 13	6th 6th 7th	Nine general education teachers; 2 special education teachers	Behavioral intervention, target behavior – task engagement during mathematics class (also conducted during two other subjects for each student including language arts, reading

								class, social studies, sciences)
Collins, Hager, & Galloway (2011)	X		Morgan: Low	Special education resource classroom	14	Middle school grade n/a	1 special educated teacher, current a master in special educated student with 3 years classroom experience, 1 paraprofessional	Order of operations (addition and multiplication) embedded in teaching computation of sales tax using prices in newspaper ads; Task analysis: Enter amount of item, press X, press 6, press %, press M+, press +, press = Then select correct choice presented on answer paper
Fletcher, Boon, & Cihak (2010)	X	X	Robert: Low Ken: Low	Special education classroom	13 14	7^{th} 8^{th}	Special education teacher with three years experience in a self-contained special education class working with students with mild to moderate disabilities; Paraprofessional with four years experience in early education classroom and one year in self-contained special education classroom	Single-digit addition problems, using *TOUCHMATH* program and a numberline
Gardil & Browder (1995)		X	Sarah: Low	Part time special education; Part time general	12	Middle school	Special education and support aide, details/experience n/a	Money selection for purchasing, selected from wallet with mixed money

				education with instructional support		grade n/a		amounts; taught response class discrimination using 10 discrete trials, generalised to untrained novel items and community purchasing context (7-11 or vending machines)
Holifield, Goodman, Hazelkorn, & Heflin (2010)	X		Tony: Low Graham: Low	Public elementary school; self-contained class for students with Autism	10 9	Eleme ntary grade n/a	Teacher was second author (PhD)	Attending to task and academic accuracy; Tony 20 questions, Graham 15 questions; Used self-monitoring, attending required: reading or writing on mathematics worksheet, counting manipulatives, erasing an answer, following teacher directions, asking or answering task related questions
Jimenez & Kemmery (2013)	X	X	Eric: Low Jon: Low	Elementary school; self-contained special education classroom for students with autism	n/a	4th 4th	Special education teachers; average of 8 years experience working with moderate-severe students; all had Bachelor in Special Education, one purusing masters	Early Numeracy Curriculum for students with moderate to severe intellectual disability (Jimenez, Browder & Saunders, 2013) to teach: Counting 1-5; Sets 1-3, 1-4; Add premade sets with sum to 5; Symbols =,>; Patterns ABAB; Measurement 1-5;

								Cakebdar dates 1st - 10th, read two weeks ahead, Number identification 1-10
Jowett, Moore, & Anderson (2012)	X	X	Jack: Low	ABA therapy, 3 hrs 5 days/week & kindergarten 5 days/week with ABA therapist support; Intervention in kindergarten classroom with 20 other children and 4 teachers	5	Kindergarten (Australian system)	Two ABA therapists	Counting (Angry Birds, using video self-modeling): identify, write and comprehend quantity of numbers 1-7
Kamps, Locke, Delquadri, & Hall (1989)	X	X	Student 1: Low Student 2: Low	Elementary special education classroom for children with autism	9 11	Elementary, grade n/a	Special education teachers, experience not specified, 5th grade peer tutors	Recognize the value of money; penny, nickel, dime, and quarter; (also expressive language, oral reading, comprehension)
Leaf, Sheldon, & Sherman (2010)	X		Brady: insufficient information provided	Research room	5	Grade n/a	Specialist intervention research therapists	Simple addition (single digit + single digit)
Legge, DeBar, & Alber-Morgan (2010)	X		Adam: Joshua: Insufficient information provided	Self-contained special education classroom; General education classroom and special education supplemental	13 11	6th 5th	Intervention therapist, teaching assistant, primary researcher; education and experience not described	Attending to task during mathematics, using momentary time-sampling measurement Study did not collect data on academic productivity (completion and accuracy

				instruction in mathematics (50 mins/day); Data collection in special education classrooms				of mathematics assignments)
Levingston, Neef, & Cihon (2009)	X	X	Matt: diagnosis of autism, detail n/a	General education	10	4th	Pursuing Master in Education	Word problems: identification of label, operation, larger number, smaller number
McEvoy & Brady (1988)	X	X	Micki: Low Sandy: Low	Special education class for severe behavior disordered children	9 7	Read-ing level: 2nd 2-4 years below grade	Special education teacher and one aide (education and experience not described)	Single digit addition problems; Matching pictures to numerals 1 and 2
Morrison & Rosales-Ruiz (1997)	X		Male participant: Low	Home taught	5	n/a	Behavioral therapist (education and experience not described)	Counting using objects
Neely, Rispoli, Camargo, Davis, & Bowles (2013)	X		Elton: High Dan: Low	General education; Early childhood centre for autism	7 3	3rd Early Interve ntion	1st year doctoral student with 2 years experience; 3rd year doctoral student with 3 years experience	Double digit subtraction; Matching colour cards Target skill had been mastered prior to intervention
Polychronis, McDonnel, Johnson, Riesen, &	X	X	Jason: High	General education	7	2nd	One of four teachers had special education license; One additional teacher had	Telling time (15 mins, 30 mins past hour)

Jameson (2004)							attended special education workshop	
Rapp, Marvin, Nystedt, Swanson, Paananen, & Tabat (2012)	X		Grant: High Larry: High Jeff: High	General education; Intervention conducted in separate classroom	9 8 7	Grades n/a	Intervention conducted by undergraduate and graduate students in Behavior Analysis program	Addition, subtraction (with and without borrowing); Single digit multiplication; Addition: left to right and top to bottom presentations ie 3 x 2 = $\begin{array}{r}3\\ \boxed{\times\ 2}\end{array}$
Rockwell, Griffin, & Jones (2011)	X	X	Samantha: Low	General education with mathematics in resource room four days/ week	10	Very low; entering 4th	Intervention conducted by first author	Addition, subtraction word problems 12 apples and 15 oranges = 27 pieces of fruit Get more additive change Get less subtractive change, Compare larger and smaller amounts
Shaefer-Whitby, Travers, & Hamik (2013)	X		Nick: High Nate: High Chris: High	General education	14 13 13	8th 7th 7th	Certified exceptional educator provided school support services, intervention involved individual teaching	Florida curriculum (FCAT) based measures of one-, two-, and three-step word problems; 6th grade FCAT questions for generalisation to general education classroom
Tiger, Bouxsein, & Fisher (2007)	X		Joe: n/a	Day treatment centre as educational placement in	19	n/a	Therapist, education and experience not described	Multiple choice format, Easy problems: single and double digit multiplication, single and

				jeopardy due to behavioral problems				double digit addition, exponential powers; Medium problems: long division; Difficult problems: Geometry, trigonometry, calculus Intervention targeted response time latency (< 5 secs) requiring discrimination between quick and long completion time of problems
Waters & Boom (2011)	X		Trent: Low Alex: Low	Special education classroom for all academics	15 16	9^{th} 10^{th}	Special education teacher, researcher, education and experience not described	Subtraction of 3 digit money computational problems using regrouping
Weng & Bouck (2014)		X	Mitch: Low Manny: Low Leo: Low	Special education	15 15 15	11^{th} 8^{th} 8^{th}	Special education teachers; Interventionist, doctoral student with several years of experience working with students with autism, trained second observers (2 special education teachers, 2 paraprofessionals)	Price comparison: independently selecting the lowest priced item
Yakubova, Hughes, &	X		Jeff: Low Brian: High	General education	17 19	Grade n/a	Special education teacher and research	Accuracy of problem solving performance on

Hornberger (2015)			Kevin: Low	secondary school, performing below grade level; mathematics conducted in special education class, intervention in separate location	18	set of five word problems involving subtraction of fractions with unlike denominators (including mixed numbers)
	26	14				

In a best evidence synthesis, King et al. (2016) identified 14 studies that met the quality assessment guidelines for single-subject research described by the What-Works Clearinghouse. Eleven research teams reported findings for a total of 28 participants. Participants included 20 males and eight females, within the age range five to 17 years. While the WWC standards are widely used, debate continues to surround guidelines for analysing intervention effects with SCD research (Horner, Kratochwill, 2012; Kratochwill et al., 2013; Scruggs, Mastropieri, 2013; Carr et al., 2014).

Using alternative quality assessment guidelines, Gevarter et al. (2016) adopted The Evaluative Method for Determining Evidence-Based Practices in Autism developed by Reichow, Volkmar, and Cicchetti (2008) to assess the strength of evidence for each study identified in the systematic literature review. Gevarter et al. identified 22 studies that targeted improvement in mathematical accuracy and seven studies that targeted engagement in mathematics (three of which were also included in the mathematical accuracy studies). In total, 26 studies reported findings for a total of 53 participants.

The publication trend of empirical mathematics intervention research conducted with students with ASD was examined over time. While King et al. (2016) reported upon 14 studies, and Gevarter et al. (2016) reported on 26 studies, 11 studies were identified as common to both reviews. Accordingly, noting that 11 of the 40 studies were duplicates, a total of 29 unique studies were identified. The publication data was examined in a line graph, and a surge in publication volume has been observed in recent years. Figure 1 illustrates the publication volume over time for the studies included in the systematic reviews of King et al. (2016) and Gevarter et al. (2016).

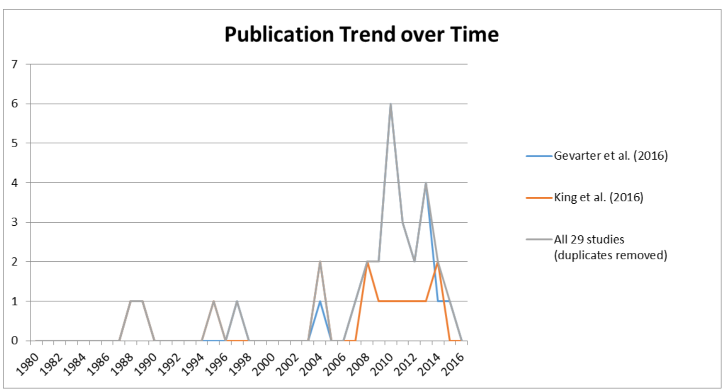

Fig. 1. Publication Trends of Empirical Intervention in Mathematics for Students with Autism Spectrum Disorder

Drawing from the dataset developed by King et al. (2016), these authors reported the prominent use of prompting and consequence-based procedures for students with severe cognitive disabilities. For higher functioning students, approaches associated with achievement gains such as representation techniques were reported in fewer cases. While school placement was not reported for 35% of participants, approximately 39% of participants received mathematics education in special education settings and 25% in general education settings. King et al. importantly noted that maintenance and generalisation measures were reported for only 30% of studies.

Gevetar et al. (2016) reported that most of the studies that targeted an improvement in mathematical accuracy that had been successful included both behavioral and academic components. Further, they noted that the majority of research had targeted teaching foundation mathematics skills to participants with ASD and a co-occurring intellectual disability. It was reported that the majority of participants had received intervention in restricted contexts such as one-on-one instruction or a self-contained classroom. Only a limited amount of research had included high functioning individuals and/or examined more complex mathematical skills such as word problems. A paucity of research teaching more complex skills was reported.

The intervention features describing participant cognitive functioning, daily classroom setting, and teacher training and experience with supporting students with ASD in their mathematics learning are summarised in Table 2.

Table 2. Mathematics Intervention Summary Statistics

Participant	High-Functioning		Low-Functioning		Insufficient detail	
	16	27.1%	36	61.0%	7	11.9%
Mathematics Classroom Setting	General Education		Special Education		Home School	
	18	30.5%	40	67.8%	1	1.7%
Mathematics Teacher Description	General Education Teacher		General Education Teacher plus Aide		Special Education Teacher	
	5	8.5%	6	10.2%	48	81.3%

Thirty-six of 59 students (61.0%) were described as low-functioning, and comprise the majority of students included in mathematics interventions. Sixteen students (27.1%) were described as high-functioning, and insufficient detail was provided for seven students (11.9%).

The majority of students, 40 of 59 (67.8%), received mathematics lessons in a special education classroom setting. Eighteen students (30.5%) remained in their general education classroom for mathematics lessons. Only one study included a home schooled student (1.7%). Fourty-eight students (81.3%) received mathematics instruction from a special education teacher, and a further six students (10.2%) had a teachers' aide present to assist the general classroom teacher. Five students (8.5%) were taught mathematics by their general education teacher.

Table 3. Mathematics Intervention Skills Targeted

Skill	**Number of Interventions**	**Percentage of Interventions**
Addition	6	17.1%
Subtraction	5	14.3%
Money handling	5	14.3%
Word problems	4	11.4%
Counting	3	8.6%
Naming numerals	2	5.7%
Matching	2	5.7%
Multiplication	2	5.7%
Time	2	5.7%
Calendar	1	2.9%
Price comparison	1	2.9%
Using Calculator	1	2.9%
Estimating correct solution	1	2.9%

Note. Three studies targeted task engagement as opposed to a mathematic skill

The age of students ranged from three to 19 years, with classroom placement ranging from Early intervention to 11^{th} Grade. A variety of foundational mathematics skills were targeted in the interventions. Addition was included in six studies (17.1%) and represented the most frequently targeted skill. Subtraction was included in five studies (14.3%). Applications of basic mathematics skills were frequently targeted, with money handling targeted in five interventions (14.3%), and word problems in 4 interventions (11.4%). Counting was targeted in three interventions (8.6%), naming numerals in two interventions (5.7%), matching in two interventions (5.7%), multiplication in two interventions (5.7%), and telling time in two interventions (5.7%). Using a calendar, comparing prices, using a calculator and estimating a correct solution were each targeted in one intervention respectively (2.9%). Conceptually separate, the ability to engage in the task at hand formed the target behavior in three studies.

6. Discussion

The current body of intervention research targeting skill acquisition, or performance improvements in mathematics skills is largely reflective of low-functioning students who are studying mathematics with special education teachers or with a teachers' aide. The majority of the studies have primarily targeted basic mathematical skills. Educating this group of students appears to be resource intensive, with the majority of teachers requiring additional training in special education or enlisting the additional support of an aide.

While ten studies included high-functioning students (*Adcock, Cuvo, 2009; *Banda, Kubina, 2010; Burton et al., 2013; Cihak et al, 2010; Neely et al, 2013; Polychronis et al., 2004; Rapp et al., 2012; Schaefer, Whitby, 2013; Travers, Hamik, 2013; Yakubova et al, 2015) it appears that there is a paucity of research being conducted with high functioning students, or with interventions that are targeting complex mathematical skills.

Elsewhere in the literature, staff training within the field of autism services has been explored to better identify causes of frustration that may cause conflict in the teaching and learning of mathematics. Dillenberger, McKerr, Jordan, and Keenan (2016) reported that teachers are not offered training in autism or appropriate inteventions. Responses to their survey indicated that although all teachers (n=43) had taught children with autism, none of whom had received autism training before they started working with children with this diagnosis. The participants in the survey were reportedly aware of the subsequent stress and potential to burn-out as a result of undertaking a job in which they were ill-prepared for.

However, there are several important reasons to pursue mathematics education research with higher functioning students on the autism spectrum. History has suggested that many significant contributions in the fields of mathematics and science have been made by individuals such as Albert Einstein, Isaac Netwon and Paul Dyrek, who are believed by experts to fit the diagnosis of ASD. Accordingly, many advocate the importance of giving individuals on the autism spectrum a voice in the research.

Consolidating these arguments, it is possible that adopting a values based paradigm to further our understanding of mathematics teaching and learning for students with ASD may prove beneficial.

7. Values and valuing in mathematics education

According to Authors (in press), valuing refers to an individual's embrace of convictions which are considered to be of importance and worth. It provides the individual with the will and grit to maintain any 'I want to' mindset in the learning and teaching of mathematics. In the process, this conative variable shapes the manner in which the individual's reasoning, emotions and actions relating to mathematics pedagogy develop and establish.

Valuing as a driving force affects cognitive and affective aspects of the tasks being valued. A student might, say, value *problem-solving.* This valuing would thus mean that the student would develop a positive attitude to problem-solving situations, either at home or outside. She would feel engaged to any task in real-life which requires problem-solving. On the cognitive side of things, the valuing of *problem-solving* would drive her to learn about problem-solving strategies, or to take part in discussions, for examples.

In other words, as we advocate for a broader scope of mathematics education research with ASD students (such as working with high-functioning students or more complex mathematics tasks), we acknowledge the increased complexity of the research field. We are concerned that a research paradigm which continues to adopt the cognitive perspective to the educational needs of all ASD students might be too restrictive. That is, in comparing the pedagogical needs and demands of high-functioning and other ASD students and of their mathematics teachers, we want to avoid developing a focus on differences in cognitive reasoning skills only. Prior research such as Pitten (2008) serves to highlight this concern, and thus the integration of the values approach to any existing autism research paradigm would represent an empowering interpretive lens for all ASD students.

8. Conclusion

This systematic literature review has confirmed growing interest in the research of mathematics education for students with a diagnosis of ASD. However, despite this surge in publications, little is known about teaching higher ability mathematics students, and in particular about teaching more complex mathematics topics. Our current findings indicate that ongoing research to address this gap so as to further our understanding in these areas is highly warranted. We have suggested the suitability of adopting a values based paradigm to contribute to the field by advancing a deeper and more correct understanding of not just the psychology but also other aspects of teaching and learning mathematics to increasing numbers of diverse ability learners on the autism spectrum. By examining the role of values, and the influence they may exert over behavior, findings from future research may contribute to the facilitation of improvements in mathematics teaching, and learning outcomes for this growing population of high potential learners.

We see merit in furthering the understanding of mathematics teaching and learning for this population and suggest the following foci as elements of future research:

1. Research that explores ASD student and teacher valuing in mathematics classrooms
2. Research that compares/contrasts ASD student valuing to their typically developing peers
3. Research that compares/contrasts teacher valuing in special education and general education settings

*References marked with an asterisk indicate empirical intervention studies included in the review.

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